Thermalization in the Early Universe

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Abstract

The formation of the present-day universe is one of the most acitve research topics on the meeting point of particle physics and cosmology. According to recent theories a sequence of symmetry breaking phase transitions has driven our universe to its present state. These transitions were preceeded by an equilibration stage starting directly after inflation. This stage (called reheating) is the actual topic of the talk. A survey of new theories is presented and an insight is given to the nonequilibrium QFT. The methods applied in the study of this scenario can be useful in statistical physics: dynamical aspects of relaxation processes and phase transitions are investigated.

Introduction

Scalar field theories play a key role in several phenomena of particle physics ranging from the exponentially inflating universe through the onset of equlibrium in the early universe to the series of symmetry breaking phase transitions guiding the universe to its present state. Beyond these direct applications of the real-time scalar theories the study of their dynamics is a basic issue towards the understanding of relaxation and dynamical phase transitions in a more general context as well.

As a most convincing solution to a number of cosmological paradoxes (e.g.: flatness of universe, horizon problem, paradox of primordial monopoles) inflationary cosmology [1] introduces a scalar field with non-vanishing vacuum energy density, called *inflaton*. Due to its initial excitation at Planck-scale the metrics began to expand exponentially. The expansion, called inflation, led to an extreme dilution and cooling of the universe. After 10^{-35} seconds of expansion inflation ended in coherent oscillations of the inflaton field around the minimum of its zero temperature effective potential.

Contrary to a dilute and cold post-inflationary universe present early universe theories suggest hot "almost-thermal" initial conditions. There must have been a process that accounts for the rapid equilibration of our universe. This process, called *reheating* governed the redistribution of the energy from the out-of-equilibrium Bose-Einstein condensate of the inflaton field to the other inhomogeneous matter fields within approximately 10^{-31} seconds.

Whether the GUT-scale temperature ($\sim 10^{12}$ GeV) was reached at the end of reheating is still an open question. If so, then massive ($M_X \approx 10^{12}$ GeV) and long living ($\tau_X \approx 10^{22}$ year) decay products of the inflaton may be candidates for the source of the mysterious ultra high energy cosmic radiation (UHECR) [2].

In this talk I survey the basic non-equilibrium phenomena of scalar fields that played important role in the early universe. First, relying on recent field theoretical literature,

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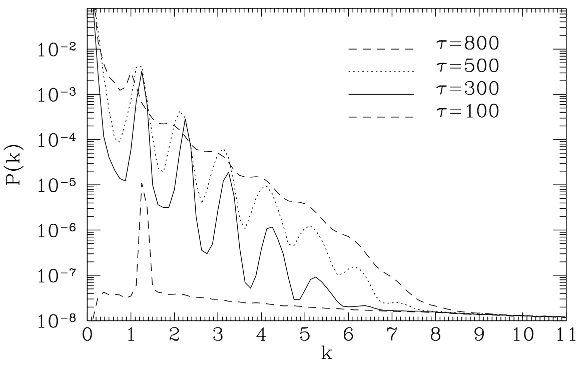

Figure 1: Evolution of the power spectrum of the decaying inflation field during the prehating of the universe [8].

the equilibration mechanism of scalar fields is discussed. Then, following my earlier work [3], I turn to the dynamics of first order phase transitions and study how a scalar field thermalized in a metastable state arrives in the stable true vacuum.

Equilibration of scalar fields

The complete decoupling of the inflaton field and the other matter fields after reheating and the correct power of the density fluctuations suggested by the theory of galaxy formation require that the coupling constant of the primordial scalar fields characterising non-linear interactions must be $\lambda \sim 10^{-12} \ldots 10^{-14}$. Rapid thermalization with such a small coupling may only occur non-perturbatively. At the moment, however, it is unknown, how to handle out-of-equilibrium quantum fields non-perturbatively.

Classical approximation. Non-perturbative analysis of out-of-equilibriuim fields are presently possible for classical fields only. Relaxation of classical fields both in post-inflationary [4] scenarios and in a more general context [5, 6] is intensively investigated in the recent field theoretical literature. The validity of the classical approximation follows from the fact that in cosmological scenarios the soft modes are highly occupied. Classical fields of increasing temperature would demand increasing counterterms as well (remember Planck's problem with the black-body radiation). This theoretical obstacle, however, does not arise until the hard modes are not populated yet. Occupation of these modes occur at the very end of reheating. One may hope that after the non-perturbative phenomena have taken place one can proceed perturbatively with the equilibration of the rapidly relaxing hard quantum modes.

Preheating. Inflation ends in homogeneous oscillations of the inflaton field. Due to the Matthieu-instability the oscillating homogeneous mode parametrically excites the inhomogeneous modes of the matter fields it is coupled to. This mechanism (*parametric resonance*) makes the occupation numbers of a few resonant modes to grow exponentially (see the corresponging power spectrum in Fig 1). Then the particles belonging to this mode scatter on each other and produce particles with other momenta as well (*nonlinear rescattering*). By this process (*preheating*) some modes will be excited and take over the energy density from the inflatons, but the universe is still far from equilibrium.

Late reheating. Equilibrium will be rached via large number of particle physics interactions. The corresponding collision terms are derived from perturbative QFT. By solving equations non-local in time, obtained from 2PI effective action thermalization of

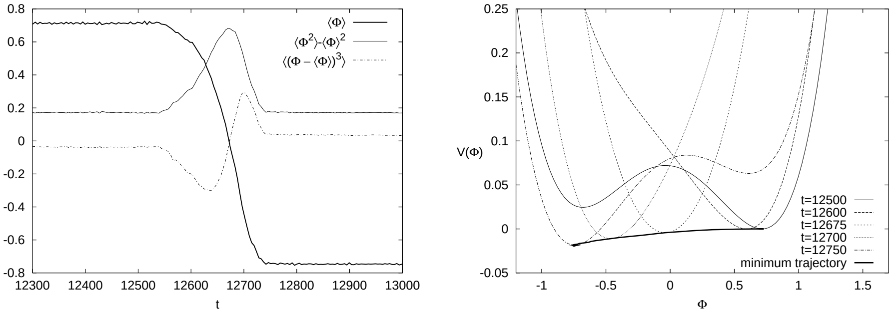

Figure 2: Homogeneous mode, its spatial fluctuation and the third cumulant of a scalar field in the course of a first order phase transition (left) and the time dependence of the instant potential governing the homogeneous mode (right).

quantum fields was demonstrated [7]. An other mechanism driving the system towards equilibrium is the scattering of the particles on a smooth low frequency background. This process is succesfully described by *linear response theory.*

Dynamical phase transitions

The range of interest of the irreversible decay of a metastable vacuum state of finite energy density covers effects from cosmological phase transitions to instabilities observed in the mixed phase of first order phase transitions of condensed matter systems [9]. Whether the relevant mechanism for a first order phase transition is the formation of bubbles of the new phase, as described by thermal nucleation theory, or the gradual change of a large region of the sample, due to small amplitude spinodal instabilities described by spinodal decomposition is also an intriguing question in heavy ion physics where the actual expansion rate of the plasma may favour one or the other scenario [10].The conventional treatment of the decay of metastable states is based on the nucleation theory but concurrent small amplitude spinodal instabilities are also present in the system. They are responsible for the flattening of the static effective potential (Maxwell cut) [11].

A numerical experiment. In ref. [3] we studied the two-stage thermalization of a classical Φ^4 theory in broken symmetry phase. Using a small external field h one of the minima of the effective potential was raised, becoming a metastable state. The field was started from this metastable state. In the first stage of thermalization the system reached a matastable equilibrium. After resting some time in this “false vacuum”, the homogeneous mode passed through the potential barrier with constant velocity and finally arrived in the “true vacuum”. This is the second stage of thermalization. Taking the spatial average of a selected field configuration and calculating its cumulants one may get an insight, what is actually happening. (see Fig. 2)

Nucleation picture. The first part of the experiment above may be easily understood in framework of the nucleation theory [12]. A system of finite temperature continuously fluctuates in every space point. It may happen that within a small area the field forms a domain of the true vacuum. The energy of this domain consists of the domain wall energy (proportional to the surface) and the negative volume energy. Small domains are often formed due to the thermal fluctuations but they collapse immediately. For domains larger than a critical volume, however, it is energetically more favourable to grow than to collapse, hence they expand until they meet an other expanding domain. After the neighbouring domains have coalesced the system arrives in the true vacuum. Thermal excitation of domains of critical size is a rare event for small values of h, but may be measured and compared to the results of the nucleation theory. We found an acceptable agreement with the theory if the nuclation rate was calculated using the finite temperature effective potential following Langer's theory [13]. Better agreement would have been achieved if the inhomogeneities of the domain wall also had been taken into account.

Dynamics of Maxwell cut. The constant velocity transition suggests the flattening of the effective potential for the period of passing to the true vacuum, and also its recovery after the trasition has been complete. To establish this picture (*dynamical Maxwell construction*) consider the equation of motion for the homogeneous mode Φ the motion of which is strongly influenced by the inhomogeneous modes $\varphi(x)$:

$$0 = \ddot{\Phi} - \Phi + \Phi^3 + 3\overline{\varphi^2}^V \Phi + \overline{\varphi^3}^V - h \equiv \ddot{\Phi} + dV_{inst}/d\Phi. \tag{1}$$

The instant potential defined in Eq. (1) is displayed in Fig. 2. Neglecting the volume of the domain walls the points of the space are separated into false-vacuum and true-vacuum domains, the former locally residing in the metastable equilibrium, the latter locally residing in the stable one. Using the fact that the system develops through local equilibrium the equation of motion for the homogeneous mode $\Phi(t)$ goes over into

$$\ddot{\Phi}(t) + \zeta_1(t) + 3\zeta_2(t)\Phi(t) = 0, \tag{2}$$

with ζ_1 and ζ_2 being cross-correlated noise functions. The equation above implies the constant velocity transition from the metastable to the stable minimum and suggests the validity a noisy effectve theory for systems being in phase coexistence.

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